NUMERICAL METHIODS

Unit I : Solution of equations and eigen value problems

Part A

1. By Newton's method find an iterative formula to find 1/N & $\sqrt{N}$, where N is a positive integer.
2. Find the positive root of $x^3 + 5x - 3 = o$ using fixed point iteration method with 0.6 as first approximation.
3. Find inverse of A = $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$ by Gauss – Jordan method.
4. State the convergence for method of false position.
5. What type of eigen value can be obtained using power method.
6. How is the numerically smallest eigen value of A obtained.
7. State two difference between direct and iterative methods for solving system of equations.
8. Give an example of (a) algebraic (b) transcendental equation.
9. Mention the two methods to solve the equation which is either, algebraic or transcendental.
10. State the formula for the method of false position to determine a root of f(x) = 0.
11. State the formula for the method of Newton – Raphson method to determine a root of f(x) = 0.
12. Write the iterative formula to find the reciprocal of a given number N by Newton – Raphson method.
13. Write the iterative formula to find the Square root of a given number N by Newton – Raphson method.
14. What is the order of convergence in Newton – Raphson method, Regula – Falsi method and Iteration method.
15. What is the condition for convergence of the iteration method for solving $x = \varphi(x)$.
16. What is the criteria of convergence in Newton – Raphson method.
17. Using Newton's method find the root between 0 and 1 of $x^3 = 6x - 4$, performing two iterations.
18. In Newton – Raphson method, the error at any stage is proportional to the ______________ of the error in the previous stage.
19. If f(x) is continuous in (a, b) and if f(a) f(b) < 0 then f(x) = 0 will have atleast ______________
20. Does a root of $x^3 - 3x^2 + 2.5 = 0$ lie between 1.1 and 1.2 ?
21. What are the direct methods to solve the simultaneous equations?
22. What are the indirect methods to solve the simultaneous equations?
23. Explain Gauss elimination method to solve the simultaneous equation.
24. Explain Gauss – Jordan elimination method to solve the simultaneous equation.
25. By Gauss elimination, Solve x + y = 2; 2x + 3y = 5.
26. Compare Gauss elimination and Gauss – Jordan method for solving simultaneous equation.
27. Gauss – Seidel & Gauss – Jacobi method always converges – Say true or false.
28. State the condition for convergence of Gauss – Jacobi and Gauss – Seidel method.
29. Why Gauss – Seidel method is better than Gauss – Jacobi method.

30. Compare Gauss – Jacobi and Gauss – Seidel method for solving simultaneous equation.
31. What type of Eigen value can be obtained using power method?
32. From the following initial Eigen vectors $(1,1)^T$, $(1,0)^T$, $(0,1)^T$, which one is the most suitable to find the largest Eigen value of the matri $\begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ in one iteration.
33. Define Eigen value and Eigen vector
34. Find the dominant Eigen value of A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ by Power method.

Part B

1. Find the real root of the equation $\log_e x = \cos x$.
2. Find the real root of $xe^x - 3 = 0$ by regula - falsi method.
3. Using the iterative method, solve $x\,e^x = \cos x$, correct to three places of decimals.
4. Using Newton – Raphson method, solve $x \log_{10} x = 12.34$. Start with $x_0 = 10$.
5. Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three decimal places.
6. Find a positive root of $f(x) = x^3 - 5x + 3 = 0$, using Newton – Raphson method.
7. Find a real root of $\cos x = 3x - 1$ correct to 3 decimal places by using Iteration method.
8. Using Regula – Falsi method, Solve $x \log_{10} x - 1.2 = 0$.
9. Solve by Gauss elimination method $3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$.
10. Using Gauss – Jordan method, solve the following system of equations $2x - y + 3z = 8$; $-x + 2y + z = 4$; $3x + y - 4z = 0$.
11. Solve by Gauss – Seidel method $28x + 4y - z = 32$; $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$.
12. Using Gauss – Seidel method, solve the following system. Start with $x = 1$, $y = -2$, $z = 3$. $x + 3y + 52z = 173.61$; $x - 27y + 2z = 71.31$; $41x - 2y + 3z = 65.46$.
13. Solve the given system of equations by using Gauss – Seidel iteration method $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$.
14. Solve the following system by Gauss – Seidel method $28x + 4y - z = 32$; $x + 3y + 10z = 24$; $2x + 17y + 4z = 35$.
15. Solve $27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$ by Gauss – Seidel method of iteration.
16. Determine the largest eigen value and the corresponding eigen vector of the matrix by power method

    (i) $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 6 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{pmatrix}$
17. Using Gauss – Jordan method, solve the following system of equations $2x - y + 3z = 6$; $-x = 2y + z = 4$; $3x + y - 4z = 0$.
18. Solve by Gauss – Jacobi method, the following equations $4x_1 + x_2 + x_3 = 6$; $x_1 + 4x_2 + x_3 = 6$; $x_1 + x_2 + 4x_3 = 6$.

19. Find the inverse of the following matrix by Gauss – Jordan method.

(i) $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ (ii) $\begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$

(v) $\begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$ (vi) $\begin{pmatrix} 8 & -4 & 0 \\ -4 & 8 & -4 \\ 0 & -4 & 8 \end{pmatrix}$

Unit II : Interpolation and Approxiamtion

Part A

1. Construct a linear interpolating polynomial given the points (x_0,y_0) and (x_1,y_1).
2. Obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula.

X :	0	2	4	6
Y :	-3	5	21	45

3. Obtain the divided difference table for the following data.

X :	-1	0	2	3
Y :	-8	3	1	12

4. Find the polynomial which takes the following values.

X :	0	1	2
Y :	1	2	1

5. Define forward, backward, central differences and divided differences.
6. Evaluate Δ^{10} (1-x) (1-2x) (1-3x)--------(1-10x), by taking h=1.
7. Show that the divided difference operator Δ is linear.
8. State the order of convergence of cubic spline.
9. What are the natural or free conditions in cubic spline.
10. Find the cubic spline for the following data

X :	0	2	4	6
Y :	1	9	21	41

11. State the properties of divided differences.
12. Show that $\Delta^3_{bcd}\left(\frac{1}{a}\right) = \frac{-1}{abcd}$.
13. Find the divided differences of $f(x) = x^3 + x + 2$ for the arguments 1,3,6,11.
14. State Newton's forward and backward interpolating formula.
15. Using Lagranges find y at x = 2 for the following

X :	0	1	3	4	5
Y :	0	1	81	256	625

1. Write down Newton's forward and backward difference formulae.
2. Write down Newon's divided difference formula.
3. Write down Lagrange's interpolation formula.
4. Define interpolation and extrapolation.

1. Find the quadratic polynomial that fits $y(x) = x^4$ at x = 0 ,1, 2.
2. Write the Lagrange fundamental polynomials $L_0(x)$ and $L_1(x)$ that satisfy the condition $L_0(x) + L_1(x) = 1$ for the data $(x_0, f(x_0))$, $(x_1, f(x_1))$.
3. Given (x_0,y_0), (x_1,y_1), (x_2,y_2) write the Lagrange's interpolation formula.
4. Construct a linear interpolating polynomial given the points (x_0,y_0), (x_1,y_1).
5. find the third divided difference of f(x) with arguments a, b, c, d with f(x) = (1 / x).
6. Show that the divided differences are symmetrical in their arguments.
7. Obtain a divided difference table for the following data :

x:	5	7	11	13	17
y:	150	392	1452	2366	5202

8. Obtain the interpolation quadratic polynomial for the given data by using Newton's forward difference formula.

x :	0	2	4	6
y :	-3	5	21	45

9. Use Lagrange's formula, to find the quadratic polynomial that takes these values. Then find y(2).

x	0	1	3
y	0	1	0

Part B

1. Using Lagranges interpolation formula find y(10) given that y(5) = 12, y(6) = 13, y(9) = 14 and y(11) = 16.

2. Find the missing term in the following table

   x : 0 1 2 3 4
   y : 1 3 9 - 81

3. From the data given below find the number of students whose weight is between 60 to 70.

Wt (x) :	0-40	40-60	60-80	80-100	100-120
No of students :	250	120	100	70	50

4. From the following table find y(1.5) and y'(1) using cubic spline.

   X : 1 2 3
   Y : -8 -1 18

5. Given sin 45^0 = 0.7071, sin 50^0 = 0.7660, sin 55^0 = 0.8192, sin 60^0 = 0.8660, find sin 52^0 using Newton's forward interpolating formula.

6. Given $\log_{10} 654 = 2.8156$, $\log_{10} 658 = 2.8182$, $\log_{10} 659 = 2.8189$, $\log_{10} 661 = 2.8202$, find using Lagrange's formula the value of $\log_{10} 656$.

7. Fit a Lagrangian interpolating polynomial y = f(x) and find f(5)

   x : 1 3 4 6
   y : -3 0 30 132

8. Find y(12) using Newton' forward interpolation formula given

   x : 10 20 30 40 50
   y : 46 66 81 93 101

9. Obtain the root of f(x) = 0 by Lagrange's inverse interpolation given that f(30) = -30, f(34) = -13, f(38) = 3, f(42) = 18.

10. Fit a natural cubic spline for the following data

    x : 0 1 2 3
    y : 1 4 0 -2

11. Derive Newton's divided difference formula.

12. The following data are taken from the steam table:

Temp^0 c :	140	150	160	170	180
Pressure :	3.685	4.854	6.502	8.076	10.225

Find the pressure at temperature t = 142^0 and at t = 175^0

13. Find the sixth term of the sequence 8,12,19,29,42.

14. From the following table of half yearly premium for policies maturing at different ages, estimate the premium for policies maturing at the age of 46.

Age x :	45	50	55	60	65
Premium y :	114.84	96.16	83.32	74.48	68.48

15. Form the divided difference table for the following data

x :	-2	0	3	5	7	8
y :	-792	108	-72	48	-144	-252

Unit III
Differentiation and Integration

Part A

1. What the errors in Trapezoidal and Simpson's rule.
2. Write Simpson's 3/8 rule assuming 3n intervals.
3. Evaluate $\int_{-1}^{1} \frac{dx}{1+x^4}$ using Gaussian quadrature with two points.
4. In Numerical integration what should be the number of intervals to apply Trapezoidal, Simpson's 1/3 and Simpson's 3/8.
5. Evaluate $\int_{-1}^{1} \frac{x^2 dx}{1+x^4}$ using Gaussian three point quadrature formula.
6. State two point Gaussian quadratue formula to evaluate $\int_{-1}^{1} f(x)dx$.
7. Using Newton backward difference write the formula for first and second order derivatives at the end value x = x_0 upto fourth order.
8. Write down the expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = x_0 using Newtons forward difference formula.
9. State Simpson's 1/3 and Simpson's 3/8 formula.
10. Using trapezoidal rule evaluate $\int_{0}^{\pi} \sin x dx$ by dividing into six equal parts.

Part B

1. Using Newton's backward difference formula construct an interpolating polynomial of degree three and hence find f(-1/3) given f(-0.75) = - 0.07181250, f(-0.5) = - 0.024750, f(-0.25) = 0.33493750, f(0) = 1.10100.

2. Evaluate $\iint \frac{dxdy}{1+x+y}$ by Simpson's 1/3 rule with $\Delta x = \Delta y = 0.5$ where 0<x,y<1.
3. Evaluate I = $\int_{1}^{2}\int_{1}^{2} \frac{dx\,dy}{x+y}$ by using Trapezoidal rule, rule taking h= 0.5 and h=0.25. Hence the value of the above integration by Romberg's method.
4. From the following data find y'(6)

X :	0	2	3	4	7	9
Y:	4	26	58	112	466	922

5. Evaluate $\int_{1}^{2}\int_{1}^{2} \frac{dx\,dy}{x^2+y^2}$ numerically with h= 0.2 along x-direction and k = 0.25 along y direction.

6. Find the value of sec (31) from the following data

$\theta(\deg ree)$:	31	32	33	34
Tan θ :	0.6008	0.6249	0.6494	0.6745

7. Find the value of x for which f(x) is maxima in the range of x given the following table, find also maximum value of f(x).

X:	9	10	11	12	13	14
Y :	1330	1340	1320	1250	1120	930

8. The following data gives the velocity of a particle for 20 seconds at an interval of five seconds. Find initial acceleration using the data given below

Time(secs) :	0	5	10	15	20
Velocity(m/sec):	0	3	14	69	228

9. Evaluate $\int_3^7 \frac{dx}{1+x^2}$ using Gaussian quadrature with 3 points.

10. For a given data find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1.1

X :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
Y:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

MA1251 – NUMERICAL METHODS

UNIT – IV : INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

PART – A

1. By Taylor series, find y(1.1) given y' = x + y, y(1) = 0.
2. Find the Taylor series upto x^3 term satisfying $2y' + y = x+1, y(0) = 1$.
3. Using Taylor series method find y at x = 0.1 if $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$.
4. State Adams – Bashforth predictor and corrector formula.
5. What is the condition to apply Adams – Bashforth method ?
6. Using modified Euler's method, find $y(0.1)$ if $\frac{dy}{dx} = y^2 + x^2, y(0) = 1$.
7. Write down the formula to solve 2^{nd} order differential equation using Runge-Kutta method of 4^{th} order.
8. In the derivation of fourth order Runge-Kutta formula, why is it called fourth order.
9. Compare R.K. method and Predictor methods for the solution of Initial value problems.
10. Using Euler's method find the solution of the IVP $\frac{dy}{dx} = \log(x+y), y(0) = 2$ at $x = 0.2$ taking $h = 0.2$.

PART-B

11. The differential equation $\frac{dy}{dx} = y - x^2$ is satisfied by y(0) = 1, y(0.2) = 1.12186, y(0.4) = 1.46820, y(0.6) = 1.7379 .Compute the value of y(0.8) by Milne's predictor - corrector formula.
12. By means of Taylor's series expension, find y at x = 0.1,and x = 0.2 correct to three decimals places, given $\frac{dy}{dx} - 2y = 3e^x$, y(0) = 0.
13. Given $y'' + xy' + y = 0, y(0) = 1, y'(0) = 0,$ find the value of y(0.1) by using R.K.method of fourth order.
14. Using Taylor;s series method find y at x = 0.1, if $\frac{dy}{dx} = x^2 y - 1$, y(0)=1.
15. Given $\frac{dy}{dx} = x^2(1+y)$, y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3)=1.979, evaluate y(1.4) by Adam's- Bashforth method.
16. Using Runge-Kutta method of 4^{th} order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with y(0)=1 at x=0.2.
17. Using Milne's method to find y(1.4) given that $5xy' = y^2 - 2 = 0$ given that $y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143$.

18. Given
$\frac{dy}{dx} = x^3 + y, y(0) = 2, y(0.2) = 2.443214, y(0.4) = 2.990578, y(0.6) = 3.823516$
find y(0.8) by Milne's predictor-corrector method taking h = 0.2.
19. Using R.K.Method of order 4, find y for x = 0.1, 0.2, 0.3 given that $\frac{dy}{dx} = xy + y^2, y(0) = 1$ also find the solution at x = 0.4 using Milne's method.
20. Solve $\frac{dy}{dx} = y - x^2$, y(0) = 1.
Find y(0.1) and y(0.2) by R.K.Method of order 4.
Find y(0.3) by Euler's method.
Find y(0.4) by Milne's predictor-corrector method.
21. Solve $y'' - 0.1(1 - y^2)y' + y = 0$ subject to $y(0) = 0, y'(0) = 1$ using fourth order Runge-Kutta Method.
Find y(0.2) and $y'(0.2)$. Using step size $\Delta x = 0.2$.
22. Using 4th order RK Method compute y for x = 0.1 given $y' = \frac{xy}{1 + x^2}$ given y(0) = 1 taking h=0.1.
23. Determine the value of y(0.4) using Milne's method given $\frac{dy}{dx} = xy + y^2, y(0) = 1$, use Taylors series to get the value of y at x = 0.1, Euler's method for y at x = 0.2 and RK 4th order method for y at x=0.3.
24. Consider the IVP $\frac{dy}{dx} = y - x^2 + 1, y(0) = 0.5$
    (i) Using the modified Euler method, find y(0.2).
    (ii) Using R.K.Method of order 4, find y(0.4) and y(0.6).
    (iii) Using Adam- Bashforth predictor corrector method, find y(0.8).
25. Consider the second order IVP $y'' - 2y' + 2y = e^{2t} S\text{int}$, with y(0) = -0.4 and y'(0)=-0.6.
    (i) Using Taylor series approximation, find y(0.1).
    (ii) Using R.K.Method of order 4, find y(0.2).

RAJALAKSHMI ENGINEERING COLLEGE
NUMERICAL METHODS
QUESTION BANK
UNIT-5

PART-A

1. Define the local truncation error.

2. Write down the standard five point formula used in solving laplace equation U_{xx} + U_{yy} = 0 at the point $(i\Delta x, j\Delta y)$.

3. Derive Crank-Niclson scheme.

4. State Bender Schmidt's explicit formula for solving heat flow equations

5. Classify $x^2 f_{xx} + (1-y^2) f_{yy} = 0$

6. What is the truncation error of the central difference approximation of

$y'(x)$?

7. What is the error for solving Laplace and Poissson's equation by finite difference method.

8. Obtain the finite difference scheme fore the difference equations $2\frac{d^2y}{dx^2} + y = 5$.

9. Write dowm the implicit formula to solve the one dimensional heat equation.

10. Define the diagonal five point formula .

1. Solve the equqtion $U_t = U_{xx}$ subject to condition $u(x,0) = \sin \pi x$; $0 \le x \le 1$,u(0,t) =

   u(1,t) =0 using Crank- Nicholson method taking h = 1/3 k = 1/36(do on time step)

2. Solve $U_{xx} + U_{yy} =$ 0 for the following square mesh with boundary values

1 2

u_1	u_2	
u_3	u_4	

1 4

2 5

4 5

3. Solve $U_{xx} = U_{tt}$ with boundary condition u(0,t) = u(4,t) and the initial condition

   u_t (x,0) = 0 , u(x,0)=x(4-x) taking h =1, k = ½ (solve one period)

4. Solve $xy'' + y = 0$, y(1) =1,y(2) = 2, h = 0.25 by finite difference method.

5. Solve the boundary value problem $xy'' - 2y + x = 0$, subject to y(2) = 0 =y(3).Find

   y(2.25),y(2.5),y(2.75).

6. Solve the vibration problem $\frac{\partial y}{\partial t} = 4\frac{\partial^2 y}{\partial x^2}$ subject to the boundary conditions

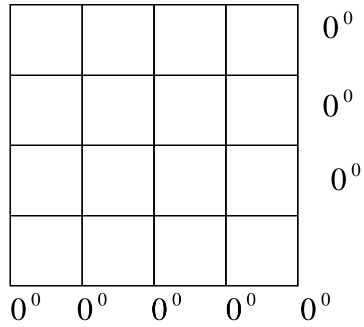
   y(0,t)=0,y(8,0)=0 and y(x,0)=$\frac{1}{2}$ x(8-x).Find y at x=0,2,4,6.Choosing $\Delta x = 2$, $\Delta t = \frac{1}{2}$ up

   compute to 4 time steps.

7. Solve $\Delta^2 u = -4(x + y)$ in the region given $0 \le x \le 4$, $0 \le y \le 4$. With all boundaries kept

   at 0^0 and choosing $\Delta x = \Delta y = 1$.Start with zero vector and do 4 Gauss- Seidal iteration.

0^0 0^0 0^0 0^0 0^0



8. Solve $u_{xx} + u_{yy} = 0$ over the square mesh of sid e 4 units, satisfying the following

conditions .

$u(x,0) = 3x \quad \text{for } 0 \le x \le 4$

$u(x, 4) = x^2 \quad \text{for } 0 \le x \le 4$

$u(0,y) = 0, \quad \text{for } 0 \le y \le 4$

$u(4,y) = 12+y \quad \text{for } 0 \le y \le 4$

9. Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$, given that u(0,t)=0,u(4.t)=0.u(x,0)=x(4-x).Assume h=1.Find

the values of u upto t =5.

10. Solve $y_{tt} = 4y_{xx}$ subject to the condition y(0,t) =0, y(2,t)=o, y(x,o) = x(2-x), $\frac{\partial y}{\partial t}(x,0) = 0$. Do 4steps and find the values upto 2 decimal accuracy.